HOW DOES THE DISTANCE BETWEEN THE LENS AND A CARD DISPLAY ON A STEREOSCOPIC VIEWER AFFECT THE SCREEN DISPARITY?

Physics Extended Essay

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TABLE OF CONTENTS

1. **INTRODUCTION**
    1.1. **Personal Engagement**
    1.2. **Research Question**
2. **WAVE BACKGROUND**
    2.1. **Superposition**
    2.2. **Refraction**
    2.3. **Diffraction**
    2.4. **Resolution**
3. **STEREOSCOPY**
    3.1. **The 3D effect**
    3.2. **Disparity**
    3.3. **Refraction in Stereoscopy**
4. **CONTROLLING PERCEIVED DEPTH IN STEREOSCOPIC IMAGES ARTICLE**
    4.1. **Main Investigation**
    4.2. **Understanding of the Investigation**
    4.3. **Formulae**
5. **EXPERIMENT PROCEDURE AND ANALYSIS**
6. **DATA AND ANALYSIS**
    6.1. **Tables**
    6.2. **Evaluation of Tables**
    6.3. **Graphs**
7. **CONCLUSION AND EVALUATION**
8. **WORKS CITED**

1. INTRODUCTION

1.1 Personal Engagement

When I first watched a 3D movie, I couldn't help but ask myself about how the 3D effect was taking place. I became more interested in this when I attended a science festival and I was totally amazed by a book with stereo cards and a stereoscopic viewer. One of the authors of the book was explaining to me how to use the stereoscopic viewer, and I really enjoyed viewing the 3D images. I remember the next few weeks looking through the pages using the viewer, but I wondered how this actually worked since there was no information about that in the book. I later learned that this was due to the superposition of two almost exact images but from different perspectives, but I then wondered: Why is it that there is always a "sweet spot" in the distance between the lens and the cards where the image is viewed perfectly? Does it have to do with the viewer? Light? The card? Is it determined, or is it just a trend in an equation of sorts? I looked back at all this when I was thinking about my Extended Essay, and my passion for physics in the area of optics and the innovations with technology using 3D in Virtual Reality.

1.2 Research Question

When a card or a display that can be viewed in 3D is seen through a 3D viewer, it will look like there is a perceived depth in the final 3D view. This is because, in the card or display, there are two similar images, but one is moved horizontally with respect to the other, and both images are superimposed by the viewer. Stereoscopy has been used in VR technology, 3D movies, 3D graphics, etc, and this technology has been applied to the areas of facial recognition, geology, chemistry, medicine, etc. (Quiroga et al.) There have been many investigations conducted about this phenomenon and its properties, and the history of stereoscopy dates back to almost 200 years when Charles Wheatstone first described it (Wheatstone et al.). A paper by Graham Jones, Delman Lee, Nick Holliman, and David Ezra called *Controlling Perceived Depth in Stereoscopic Images*, made in 2001, explored new ways to control the perceived depth with stereoscopic images, looking at the math behind the phenomena. In this Extended Essay, I will be exploring the concepts discussed in the investigation through experimental data. This is done through my research question: **How does the distance between a card and the lens affect the screen disparity?** In this EE, the screen disparity is being defined as the perceived distance between two of the same objects but viewed from the different angles of the eye. My aim is to see if the findings are in accordance with experimental data, by confirming a formula that was developed in the investigation, to hopefully provide more insight into the topic. (Jones et al.)

2. WAVE BACKGROUND

2.1 Superposition

There are many ways that waves behave that define if a wave is a wave or not. One of these behaviors is superposition. Light is a traveling wave, and all traveling waves have a maximum displacement from the equilibrium position of a wave. The principle of superposition is that whenever two or more waves intersect, the resulting displacement should be equal to the sum of the displacements of the waves. If two waves with a positive amplitude intersect, then there is constructive interference, and if two waves have opposite amplitudes, there is destructive interference and no amplitude. (Bowen-Jones et al.)

2.2 Refraction

A big factor in determining how light will go through something or how it will be projected onto something is through refraction, which is the bending of a wave while going through another medium. When light travels into another medium, there will be a change in the angle in which it is being propagated and a change in the wavelength of the wave. This happens in all kinds of directions in which the new medium is placed. However, if the medium is not flat, but curved in some way, there is another phenomena observed: the fact that an object can look upside down, extended, contracted, bigger, or smaller. The way that it is later seen has to do with the curvature of the new medium. If it is curved to the outside, or convex, there will seem to be a focus on the other side of the surface where light is incident. In other words, it will look like the light is bending to the inside. There is an opposite effect using a convex lens, a lens that curves to the inside and doesn't focus on one part, but rather looks like it bends towards the outside. (Bowen-Jones et al.)

2.3 Diffraction

Another behavior of traveling waves is diffraction. Diffraction is the bending of waves around corners or slits. In the case of stereoscopy, there are two lenses, corresponding to two slits. When light travels through slits, light bends around these slits, not going in a straight direction, but in a cone-like direction demonstrated by wavefronts. The principle of superposition can be seen when a screen is placed in front of the slits. A pattern alternating between bright and dark appears on the screen. This is the result of constructive and destructive interference happening on the screen. (Bowen-Jones et al.) An example of diffraction is shown in figure 1.

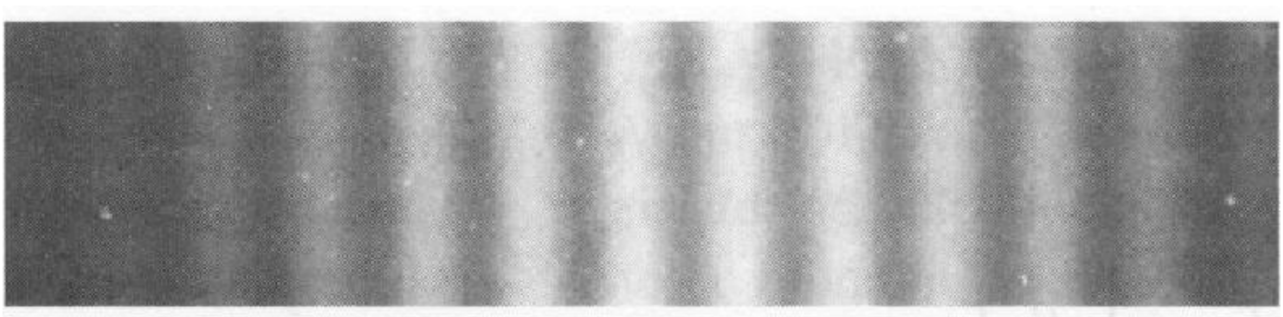


Figure 1: Double-slit diffraction pattern on a screen (Branson, Jim. "Diffraction from Two Slits." *Diffraction from Two Slits*, University of California, San Diego, 22 Apr. 2013, https://quantummechanics.ucsd.edu/ph130a/130_notes/node63.html.

2.4 Resolution

Logically, when there are two sources of light, there will be two diffraction patterns. However, if the diffraction patterns are really close together, it will seem like there was only one source of light. If this happens, it is said that the resulting image is not resolved. According to the Rayleigh criterion, the criterion that delineates what images are resolved and what images are not resolved. The Rayleigh criterion states that two sources are resolved if the principal maximum diffraction pattern is no closer than the first minimum of the other pattern. This criterion also delineates what is meant by two images looking like they are different. Two images are just resolved if they are exactly described by the criterion, and they are resolved if the first minimum of one image is farther from the first maximum of the other image. (Bowen-Jones et al.) In the case of stereoscopy, since the images should appear to be one, ironically no resolution is a goal. “No resolution” in this context means that the images should appear as one, but in a way that there should be a perceived depth, that the final image looks 3D to the viewer.

3. STEREOSCOPY

3.1 The 3D effect of Stereoscopy

A stereoscopic card has two images. These images are almost the exact same, but the difference between the images is that one of the images is moved horizontally. This is because the two eyes of the viewer see one thing from different perspectives. Zero resolution is important here because the point of 3D viewing is that you can see an image with perceived depth, so there should be only one resulting image. There is a separation between both eyes, which is taken advantage of when designing stereoscopic viewers. These stereoscopic viewers (example in figure 2 on the right) are used when seeing stereo cards. They normally have convex lenses, which focus the light from outside into the center. This is also what can make the 3D effect.

3.2 Disparity

This image separation corresponds to the length between both human eyes. This is what causes a 3D effect. The disparity measure in images is really important because an inaccurate disparity can show distortion of images and it can cause eye strain. One type of disparity is called the screen disparity. This disparity is made through the borders of what both eyes can see, and this is seen in figure 3. (How to see 3D)

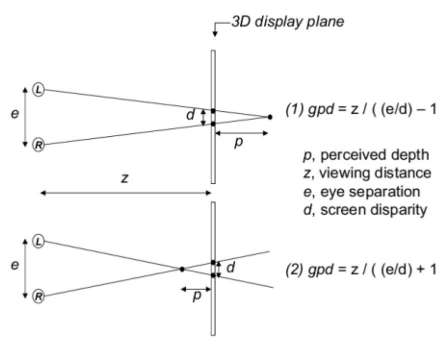


Figure 3: Disparity and other values (Holliman)

In the diagram, L and R are the left and right eyes, and the lines from the eyes to the display plane represent the viewing of both eyes. GPD represents the geometric perceived depth. The first case is the case in which the viewing lines do not intersect i.e. where the focus of the viewing does not make these lines intersect, and the second case represents where the viewing lines do intersect. For my experiment, I will be measuring the value d as shown in Figure 3. Whether the first diagram or the second diagram is used does not matter since the viewer will have the same d-value.

3.3 Refraction in Stereoscopy

In order for a stereoscopic card to be viewed, the stereoscopic viewer needs to have a double convex lens, for viewers made to view cards. The two double convex lenses focus the image to make both eyes see the image at the same time. This works through the fact that light is moving outwards like a wave, and when it comes in contact with the double convex lens, the wavefronts come closer together and then get reflected, so that the wavefronts are in the opposite direction to the curve of the lens. After that, the wavefronts move out of the lens in the same way, and since the wavefronts move out like they are moving to a center, the waves then get focused. (Bowen-Jones et al.) This is also explained in figure 4:

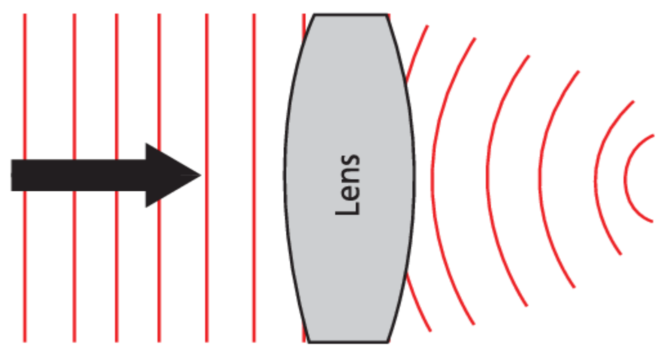


Figure 4: Wavefronts through a double convex lens (Jinabhai et al.)

In figure 4, the wavefronts are coming in through the left side of the double convex lens, and then they come out, more focused.

4. SOURCES

4.1 Sources used in the Extended Essay

The main source used in my investigation was *Controlling Perceived Depth in Stereoscopic Images*. It is a scholarly article published in the Proceedings of SPIE, The International Society for Optical Engineering, so I don't doubt the reliability, since this journal is peer-reviewed. Other sources include university research papers and physics textbooks, all of which are considered reliable sources. I used these sources only as background on the topic since my experiment was the focus of my investigation, and my focus article of the experiment was *Controlling perceived depth in Stereoscopic Images*.

4.2 Main Source

In 2001, Graham Jones, Delman Lee, Nick Holliman, and David Ezra described "a novel method for calculating stereoscopic camera parameters" with many contributions, including providing the user of a viewer intuitive controls which can relate to easily measured physical values. For the investigation, they used an OpenGL camera for doing the experiments in order to get the values needed while using trigonometry to get all relevant equations. They also used stereoscopic cards, but these are not specified. For simplicity, I used an Owl stereoscopic viewer, with no camera. Since I will not be doing any mathematical calculations of the disparities, I will have to assume that the viewer used and the Owl viewer should have similar characteristics. (Jones et al.)

4.3 Understanding of the Main Source

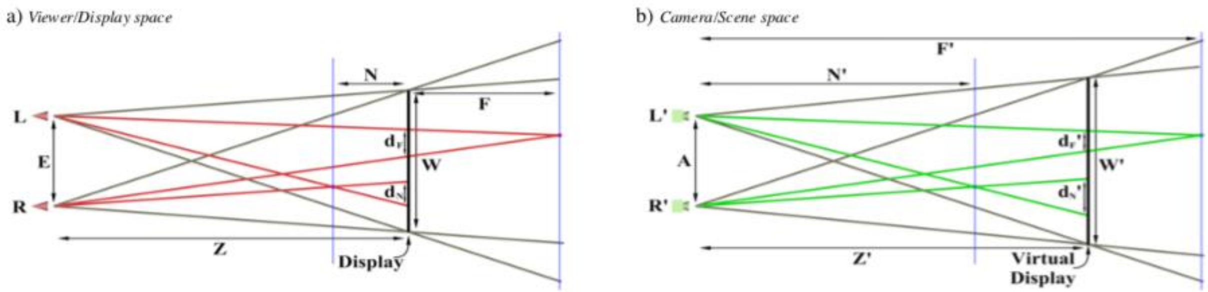


Figure 5: Variables used in *Controlling Perceived Depth in Stereoscopic Images* (Jones et al.)

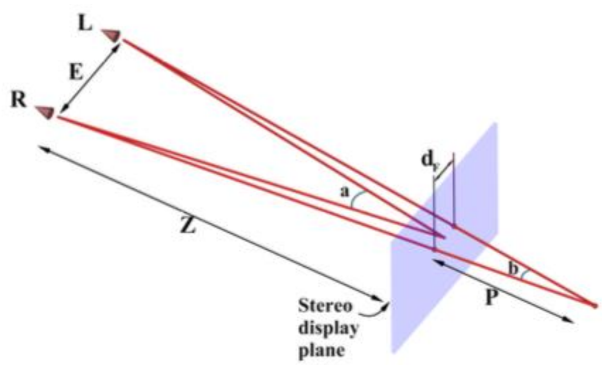


Figure 6: Figure 5 from an angle (Jones et al.)

There are two main parts to the investigation, apart from the introduction and the conclusion: *4. WHEN THE VIEWER IS STATIC*. In part 4, Jones and his team mathematically modeled the screen disparities - d_N and d_F - and their respective world disparities, where a camera is used. The formula with d_n is the formula that I want to apply to my experiment:

$$d = \frac{NE}{Z - N}$$

Since the screen and the world disparities are not going to be equal, but a proportion will be the same, d_N is divided by d_F along with their world disparities in order to get a constant R. The distance to a virtual display, $Z^/$, is then expressed in terms of values that can be measured with the camera. Then, W', the distance of the virtual display, is determined trigonometrically. A scaling factor, S, is determined as the ratio of W, the distance of the display, to W':

$$S = \frac{W}{W'}$$

Noting that d'_N, one of the world disparities, is equal to S times d_N, the value of A, a constant, is calculated. This constant is the one needed to control depth perception, which is the key to the investigation - controlling the overall depth perception to ensure minimum eye strain and distortion. This is another formula I will be trying to apply to my design. (Jones et al.)

4.4 Formulae

The initial formula I will be trying to apply is this:

$$d_n = \frac{NE}{Z - N}$$

Where d_n is the disparity, E is the distance between the lenses, which is constant, Z, which is the distance between the card and the lenses, and N, which is the distance between the minimum distance the object should appear to the viewer and the card itself. N is also constant.

For this next equation, I will assume that W' and W are approximately the same since the lens distance will be the same in my experiment, so that S = 1. Here is the final equation:

$$A = \frac{d_n \times N}{Z - N}$$

Where A is a constant, and d_n, N, and Z are all defined as before. Since S = 1, there will be no factor multiplying the equation that would still give the constant A. The equation can then be rearranged to this:

$$d_n = \frac{A(Z-N)}{N}$$ (Jones et al.)

5. EXPERIMENTAL PROCEDURE AND MATERIALS

To conduct my experiment, I will be using the Owl stereoscopic viewer, a 30cm ruler with a +/- 0.1cm error, and some construction paper to make a mock stereoscopic card, for simplicity. Two squares that are black and of the same size have to be cut out and glued onto construction paper of a contrasting color that is not necessarily white - it has to be a similar color. The resulting card has to be placed on the Owl viewer with a line through the middle of both squares, and the squares have to be 6.5cm apart, the average distance between two equal dots on a stereoscopic card used by the Owl viewer. For best results, place one of the black squares so that its center is 3.5cm from the middle, and the other one 3cm from the middle. The setup is seen on the image to the left. The procedure is this: mark on the Owl viewer the places where the lenses are 12, 13, 14, and 15 cm from the card since 12cm is the minimum and 15cm is the maximum. These distances correspond to the value Z on the equation used. At each height, look down through the lens on the stereoscopic viewer and measure the distance between images as seen through the Owl, which is the disparity. In the equation, the disparity would be d_n. The disparity can be measured if you try to view the ruler with one eye, but view the image with both, which is the way I will be measuring it. If the two images look like one, then that counts as 0. After every trial, you would have to blink and then measure again.

6. DATA AND DATA ANALYSIS

6.1 Tables

Table #1

The table below shows the disparities as a function of the four different heights and the average disparity per height, with the error in the height being 0.1cm, the error on the 30cm ruler, and the error on the disparities as the largest of the standard deviations of the disparities per height.

Height of lenses (Error = +/- 0.01cm)	Disparity 1 (Error = +/- 0.01 cm)	Disparity 2 (Error = +/- 0.01 cm)	Disparity 3 (Error = +/- 0.01 cm)	Disparity 4 (Error = +/- 0.01 cm)	Disparity 5 (Error = +/- 0.01 cm)	Average (Error = +/- 0.01 cm)	Error (cm)
12.0cm	0.05	0.10	0.10	0.10	0.05	0.08	0.01
13.0cm	0.10	0.10	0.10	0.10	0.05	0.09	0.01
14.0cm	0.10	0.10	0.20	0.10	0.10	0.12	0.05
15.0cm	0.15	0.30	0.30	0.20	0.30	0.24	0.08

Table #2

Since the first graph, based on the table above, will be linearized to fit a straight line into another graph, I felt the need to include another table with the new data. Since the next y-axis will be ln (Disparities), the error has to be divided by ten.

Height of Lenses from Card (Error = +/- 0.01cm)	Natural logarithm of Average Disparity	Error (cm)
12.0cm	-2.52	0.02
13.0cm	-2.41	0.02
14.0cm	-2.12	0.18
15.0cm	-1.43	0.27

6.2 Evaluation on Tables

The table above showed the disparities I gathered from the heights. It was logical to find no standard deviations of 0, but the standard deviations were really close to the averages because the averages are really small numbers. This is because although there is some difference, the difference is almost minuscule, but the disparity will be approximating to 0. Looking at the results in the data tables, and my own personal observations while executing the experiment, it is not so clear that the study's findings of the relationship between the disparity and the distance between the lens and the display are consistent with my observations. However, the overall trend looks like it is exponential as is seen in figure 8. (Jones et al.)

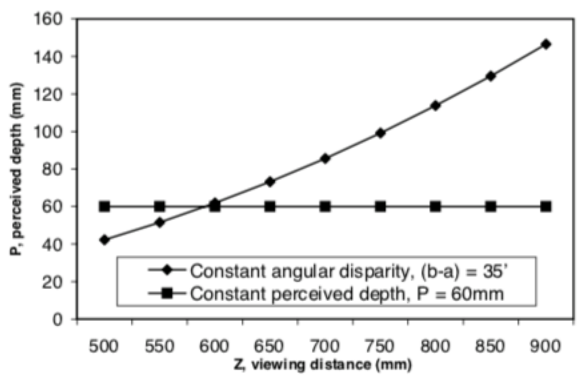


Figure 8: Graph of viewing distance vs perceived depth. (Jones et al.)

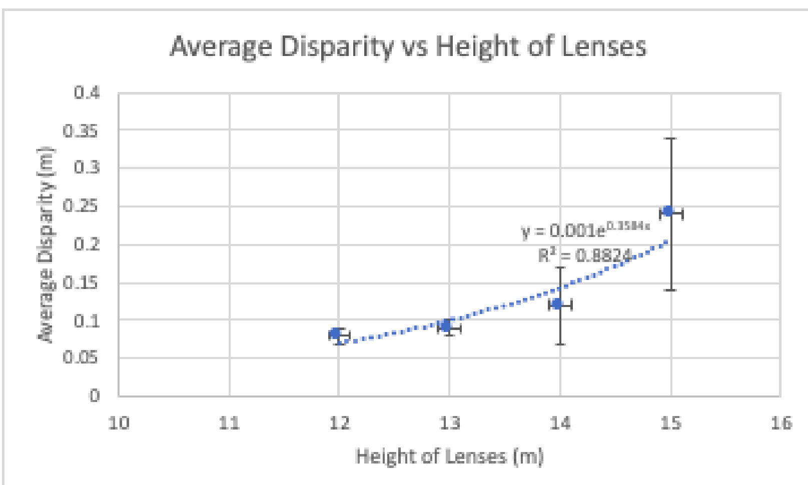
As can be seen in Figure 8, the constant angular disparity is increasing when distance increases. Since angular disparity increases in a similar trend to disparity, the overall trend is similar to my results. (Jones et al.)

It was interesting to see, however, that a polynomial to the second degree would have worked better than the exponential graph, with the polynomial trendline giving an R^2 value of 0.9849. However, polynomial trendlines are really difficult to linearize if degrees other than the highest degree of the polynomial are not equal or really close to 0. The second-degree polynomial would have helped with the second equation I was trying to apply. The inverse trendline would have also worked, but it had an R^2 value of 0.8607. This would have also worked with the first equation I was trying to apply.

On another note, I was curious about the fact that there is still a specific “sweet spot” of viewing that does not concord with the equations. The equation specifies trends, but my observations show a sweet spot, and the data doesn’t. This may be because of the fact that the data shows a trend because there is no common sweet spot for all viewers, but there seems to be a spot for each viewer, but factors like natural light can affect this, and a sweet spot might not be seen under a controlled environment.

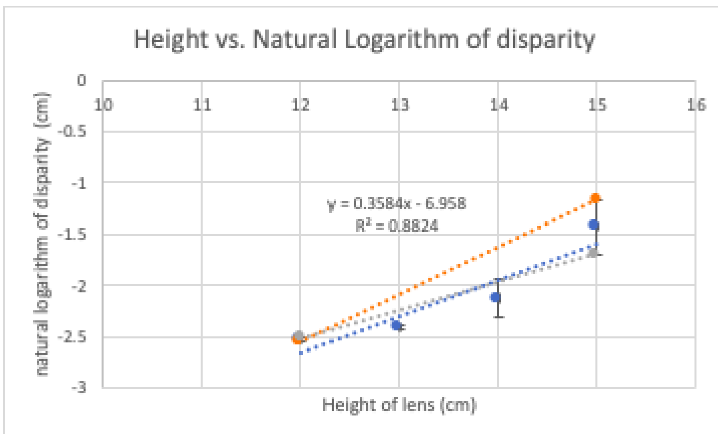
6.3 Graphs

Graph #1: Height against average disparity



Graph 1 above shows the average disparity for every distance between the lens and the card. The error on the disparity for the two largest distances has the largest standard deviations, in accordance with their respective data tables. The trend lines in the graph are of an exponential nature with an R^2 value equal to 0.8824 for the trendline. As mentioned before, a trendline of a polynomial to the second degree would have worked for one equation, and the same happens for the inverse. In order to actually determine which type of regression would be better, I would have needed more data.

Graph #2: Linearization of Graph #1



In this graph, I have linearized the data from graph 1 by plotting 1/distance on the x-axis and the same disparity on the y-axis. This time, I get an R^2 value of 0.8824, which is the same as the original graph. However, the high and the low error trendlines (high being the orange trendline and low being the gray trendline) are not really favorable for being a good fit, but it does seem like the high, low, and middle trendlines are almost intersecting at a point, although this point is near the start of the graph. I think that this is due to the fact that the uncertainty on the point of the biggest distance is the biggest, because of what I think is that if there is a lower distance, then the image will be more accurate with almost no uncertainties, and on the contrary, the bigger distances have a bigger uncertainty because of the same reason, but the reverse.

7. CONCLUSION AND EVALUATION

The aim of this Extended Essay was to explore the properties and trends of different values of importance in stereoscopy, especially distance and disparity. My research question was: how is the disparity dependent on the distance between the lens and the stereo card? I explored and answered this research question to the extent that my method allowed me to check how the disparity can vary. From Graph 1, it is evident that the disparity increases as the distance between the card and the lens increases. From Graph 2 it is also seen that the nature of this positive relationship is most likely exponential, but this doesn't clearly explain any of the equations, especially the d_n equation. Another interesting observation is that inverse and polynomial regression also fit, including the quadratic, cubic, and quartic, and the inverse and polynomial trendlines fit the equations. I believe the reason for the different trendline fittings could be the type of viewer used since the type of viewer could be a factor in how the distance is affecting the disparity. Since the relationship is most likely exponential, I can say that this doesn't support the findings of Jones et al.

A more detailed study of the viewer would provide a way to model the physical explanation for this. I could use different viewers with different properties and see what kind of graph and trend I get. The greatest deviation is that for the biggest distance, which the data shows has a greater deviation from the smallest as the distance increases. I can attribute this to the fact that this is far away from the supposed "sweet spot", where the disparity measured is reasonably close to 0, but not exactly 0. After analyzing and evaluating the data, I will say that I was able to answer my question. The answer would be that the distance affects the disparity in an exponential nature, but if more data can be collected, I can validate the results and add more insight. Due to the limited domain of the distance variable for my viewer, I wasn't able to put 5 values for the investigation, so in the future, I could also check with a larger viewer.

A strength of my experiment was the use of a black-and-white-no-design stereoscopic card because then there are no variations in the wavelength. A weakness, however, is that the precision and accuracy of my eyes and observations have human error since I could not use a camera for my experiment like how was used in the article. Variables such as the light and natural light were controlled at all times.

Potential for uncertainty comes from the accuracy of the ruler that I used, which would be a systematic error, as well as accidental push-downs on the owl stereoscopic viewer, which would be a random error. However, the data seems to show a different trend than the equation, which leads me to think that the errors affected my conclusions about the relationship between the distance between the lens and the card, and the disparity. Coincidentally, my errors are really big, and this can harm the reliability of my data.

Another limitation that I had included the fact that I did the experiment myself, and since people have different types of eyes, other people might detect different trendlines. To expand the investigation even further, I can have different people with different eye types do the same experiment, and in this way, I can get different results, so I can investigate the validity of the original data and the data from my main source by comparing this with the trendlines.

I can think of a way to expand this exploration even further: how can the disparity change between the refractive indexes of different materials of cards and of lenses? This would mean not only testing the same experiment using different materials of cards and lenses, but also through different sizes of cards, lenses, and viewers. I believe that there is also room for investigation on how the trends of the disparity versus the distance may or may not change through these different factors and also through other factors like the mass and the wavelength of the waves, since I had a constant wavelength in the experiment, and the wavelength may affect the trends. I could also use a camera like how was used in the *Controlling Perceived Depth in Stereoscopic Images* investigation.

With this conclusion, I would say that I have added some insight into the nature of binocular disparity and how the distance between the lens and the card can affect stereo viewing, being partly verified experimentally, although there have been large errors. I have shown that a change in the distance between the lens and the card does make a change in the disparity being viewed.

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